STATE ESTIMATION IN DISTRIBUTION SYSTEMS

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What has been done…

We developed and tested a State Estimation approach for Distribution Systems, as well as a sensitivity analysis to test the robustness of the method.

...has multiple applications
Outline

- Smart Distribution Systems
- Challenges in Distribution Systems
- State Estimation Overview
- Use of WLS for State Estimation
- WLS State Estimation in Distribution Systems
- Case Study – IEEE 34-Bus Test System
  - Numerical results
  - Sensitivity analysis
  - Sensitivity analysis results
- Conclusions
Smart Distribution Systems

Renewable energy resources
Distributed Generation
Smart Meters (PMUs, AMI)

Smart Grid

Quality of service
Reliability

State Estimation (SE):
Provides State of the Grid in real-time for **monitoring** and **control**:

- Online contingency analysis
- Bad data detection
- Power Flow Optimization
Challenges in Distribution Systems

Transmission System vs Distribution System

- **Meshed topology**
  - Uni-directional power flows
  - Balanced lines and loads

- **Radial topology**
  - Bi-directional power flows
  - Unbalanced lines and loads
**State Estimation Overview**

**INPUT**
Measurements
(P, Q, I, V)

**State Estimator**

**OUTPUT**
State Variables
(V, θ)

**Weighted Least Squares (WLS)**

**Advantages**
- Overdetermined System
- Performs well in presence of noise

**Disadvantages**
- Fails to reject bad data
- Sensitive to initial point

**Observability**

pseudo-measurements
Weighted Least Squares:

\[
\begin{align*}
\mathbf{z} &= \mathbf{h}(\mathbf{x}) + \mathbf{e} \\
\mathbf{r} &= \mathbf{z} - \mathbf{h}(\mathbf{x})
\end{align*}
\]

\(\mathbf{z}\) : measurement vector \hspace{1cm} \(\mathbf{e}\) : measurement errors (Gaussian distribution)

\(\mathbf{x}\) : state variables vector \hspace{1cm} \(\mathbf{h}(\mathbf{x})\) : measurement function

\(\mathbf{r}\) : residual error

\(\mathbf{W}\) : penalty factor of measurements

Optimization problem:

\[
\begin{align*}
\min_{\mathbf{x}} J(\mathbf{x}) &= [\mathbf{z} - \mathbf{h}(\mathbf{x})]^T \mathbf{W} [\mathbf{z} - \mathbf{h}(\mathbf{x})] \\
\end{align*}
\]

\[
\begin{align*}
\mathbf{H}^T(\mathbf{x})\mathbf{W} [\mathbf{z} - \mathbf{h}(\mathbf{x})] &= 0 \\
\mathbf{H}(\mathbf{x}) &= \frac{\partial \mathbf{h}(\mathbf{x})}{\partial \mathbf{x}}
\end{align*}
\]
Iterative Process:

\[ x_{k+1} = x_k + \Delta x_k \]

given that the increment \( \Delta x_k \) is given by

\[
[G(x_k)]\Delta x_k = H^T(x_k)W[z - h(x_k)]
\]

where

\[
G(x) = H^T(x)WH(x)
\]
WLS State Estimation in Distribution Systems

Measurements and State Variables vectors

\[ z = \left[ P_f^T, Q_f^T, I_l^T, V_m^T, P^T, Q^T, P_L^T, Q_L^T \right]^T \]

\[ x = \left[ V_m^T, \theta^T \right]^T \]

Jacobian Matrix of the State Equations

\[
H(x) = \begin{bmatrix}
\frac{\partial P^T}{\partial \theta} & \frac{\partial Q^T}{\partial \theta} & \frac{\partial I_l^T}{\partial \theta} & \frac{\partial V_m^T}{\partial \theta} & \frac{\partial P^T}{\partial \theta} & \frac{\partial Q^T}{\partial \theta} & \frac{\partial P_L^T}{\partial \theta} & \frac{\partial Q_L^T}{\partial \theta} \\
\frac{\partial P^T}{\partial V} & \frac{\partial Q^T}{\partial V} & \frac{\partial I_l^T}{\partial V} & \frac{\partial V_m^T}{\partial V} & \frac{\partial P^T}{\partial V} & \frac{\partial Q^T}{\partial V} & \frac{\partial P_L^T}{\partial V} & \frac{\partial Q_L^T}{\partial V} \\
\frac{\partial P^T}{\partial \theta} & \frac{\partial Q^T}{\partial \theta} & \frac{\partial I_l^T}{\partial \theta} & \frac{\partial V_m^T}{\partial \theta} & \frac{\partial P^T}{\partial \theta} & \frac{\partial Q^T}{\partial \theta} & \frac{\partial P_L^T}{\partial \theta} & \frac{\partial Q_L^T}{\partial \theta}
\end{bmatrix}^T
\]

Penalty factors matrix

\[ W_{ii} = \begin{cases} 
1 & \text{For the forecasted load} \\
10 & \text{For the actual measurements}
\end{cases} \]

\[ P_f: \text{Forecasted real injection} \]
\[ Q_f: \text{Forecasted reactive injection} \]
\[ I_l: \text{Line current measurements} \]
\[ V_m: \text{Voltage magnitudes} \]
\[ \theta_m: \text{Voltage angles} \]
\[ P_L: \text{Real bus withdrawals at load nodes} \]
\[ Q_L: \text{Reactive bus withdrawals at load nodes} \]
\[ P: \text{Real bus injections at generator nodes} \]
\[ Q: \text{Reactive bus injections} \]
We test the performance of the State Estimator in two different scenarios, including different quantity of measurements each time.

**Scenario 1 set of measurements:**
- Forecasted load with 10% of perturbation
- Power injection measurement at substation
- Power flow of lines 802-806, 824-828, 834-860, 836-862
- Line current of lines 800-802, 816-824, 860-836

**Scenario 2 set of measurements:**
- Forecasted load with 10% of perturbation
- Power injection measurement at substation
- Power flow measurements of lines 802-806, 834-860
- Line current measurements of line 816-824

24.9 kV
Radial feeder
Some single-phase laterals but mostly 3-phase
Case Study – Numerical results

Scenario 1
- Algorithm converged in 69 iterations
- Residual for the last iteration:
  \[ r = z - h(x) = 2.7494 \]
- Maximum difference between estimated and actual voltage magnitude value is 0.09 pu

✓ Results are very similar in both cases.
✓ The method results in a **feasible solution** for the estimate of the state variables.

Scenario 2
- Algorithm converged in 67 iterations
- Residual for the last iteration:
  \[ r = z - h(x) = 2.5293 \]
- Maximum difference between estimated and actual voltage magnitude value is 0.095 pu
Motivation: Test the robustness of the algorithm and sensitivity to bad quality input data.

Relative error of voltage magnitudes at each bus

\[
\text{Error} = \frac{V_{\text{estimate}} - V_{\text{actual}}}{V_{\text{actual}}} \times 100\%
\]

Simulation of bad quality data recreated in 4 different cases:

- **Case 1**: Increased the line power flow measurements by 2.5%
- **Case 2**: Increased the line power flow measurements by 10%
- **Case 3**: Increased the power flow and line current measurements by 2.5%
- **Case 4**: Increased the power flow and line current measurements by 10%
Case 1: Increased the line power flow measurements by 2.5%

- Voltage magnitude error for Scenario 1
  - Max. Error = -0.71%

- Voltage magnitude error for Scenario 2
  - Max. Error = -0.56%
Case 2: Increased the line power flow measurements by 10%

- Voltage magnitude error for Scenario 1
- Voltage magnitude error for Scenario 2

Max. Error = -2.85%

Max. Error = -2.26%
Case 3: Increased the line power flow and line current measurements by 2.5%

- Voltage magnitude error for Scenario 1
  - Max. Error = -1.09%

- Voltage magnitude error for Scenario 2
  - Max. Error = -1.03%
Case 4: Increased the line power flow and line current measurements by 10%

- Voltage magnitude error for Scenario 1

  ![Graph showing voltage magnitude error for Scenario 1 with max error of -4.46%]

- Voltage magnitude error for Scenario 2

  ![Graph showing voltage magnitude error for Scenario 2 with max error of -4.17%]
Summary of Sensitivity analysis results

- Absolute error in the results:

<table>
<thead>
<tr>
<th>Error Introduced</th>
<th>Scenario</th>
<th>+2.5%</th>
<th>+10%</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Power Flow Measurements</strong></td>
<td>1</td>
<td>0.71%</td>
<td>2.85%</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>0.56%</td>
<td>2.26%</td>
</tr>
<tr>
<td><strong>Power Flow and Current Measurements</strong></td>
<td>1</td>
<td>1.09%</td>
<td>4.46%</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>1.03%</td>
<td>4.17%</td>
</tr>
</tbody>
</table>
Conclusions

- SE is a powerful tool that has been traditionally used in Transmission Systems. Its application for Distribution Systems is feasible today and would enhance grid operation and planning.

- The traditional approach to this method, the WLS algorithm, can be implemented to Distribution Systems taking into account the specific characteristics of these systems.

- The tool created based on WLS algorithm showed encouraging results when applied to different Scenarios of the IEEE 34 Bus Test System.

- This algorithm is robust and still present good results under the “bad quality data” simulation.

- Application to a real feeder is currently under study, as well as other possible approaches to the State Estimation problem.
Thank you!